C ODEBREAKER

Class discussion
QUESTION 1
Strategies for codemaker
What did you do as codemaker?

• When you were playing as codemaker, what strategy did you use to pick a code?
QUESTION 2

number of codes
How many codes are there?

- Anyone have a guess how many different possible codes there are?
Number of codes

- If we used just 1 digit, there would be 6 possible codes.
What if there are 2 digits?

If 1st digit is 1:
- 6 possible choices for 2nd digit

6 possibilities for first number
- 6 possibilities for second number
  - $6 \times 6 = 36$ possible combinations

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td>25</td>
<td>26</td>
</tr>
<tr>
<td>31</td>
<td>32</td>
<td>33</td>
<td>34</td>
<td>35</td>
<td>36</td>
</tr>
<tr>
<td>41</td>
<td>42</td>
<td>43</td>
<td>44</td>
<td>45</td>
<td>46</td>
</tr>
<tr>
<td>51</td>
<td>52</td>
<td>53</td>
<td>54</td>
<td>55</td>
<td>56</td>
</tr>
<tr>
<td>61</td>
<td>62</td>
<td>63</td>
<td>64</td>
<td>65</td>
<td>66</td>
</tr>
</tbody>
</table>
What if more than 2 digits in code?

- 6 possibilities if only 1 digit
- $6 \times 6 = 36 \ (6^2)$
- $6 \times 6 \times 6 = 216 \ (6^3)$
- $6 \times 6 \times 6 \times 6 = 1296 \ (6^4)$ – Mastermind

So there are more than 1000 possible codes!
What if valid numbers in code changes?

Assume that instead of 1-6, each number can just be 1-4
• 4 if only one digit
  • $4 \times 4 = 16$ ($4^2$)
  • $4 \times 4 \times 4 = 64$ ($4^3$)
  • $4 \times 4 \times 4 \times 4 = 256$ ($4^4$)

Or if each number could be 1-10
• $10 \times 10 \times 10 \times 10 = 10,000$ ($10^4$)
QUESTION 3

Codebreaker strategy
What’s a good strategy for the codebreaker?

- How would you choose your first guess?

- What would you do next?
A possible strategy

Start with a code that has only 2 digits
  e.g., 4455, 1122, 2266

Let’s walk through some examples to see why this might be good.

Assume the code is 1231
Code is 1231 – Guess 1

• First guess is 4455

• Response is 0-0

• What do we know?

  • We should never guess a 4 or a 5 again!
Code is 1231 – Guess 2

• We know we should not guess a 4 or a 5.
• Let’s try 1122

• Response is 1-2

• What do we know?
  • One correct digit in right location, two correct digits in wrong location
  • Total of three correct digits
  • The code must include either two 1s or two 2s
  • The code must also include either a 3 or a 6 OR one more 1 or 2 (e.g., could be a code like 1211 or 2221)
Code is 1231 – Guess 3

- What should we do now?
  - There are various options that are valid
  - We might try other codes to see whether 3 or 6 are included
  - We might try to rearrange numbers to figure out which digit was in the correct location, and move others in the correct location

- Let’s try 3333
  - The response is 1-0. So we know the code contains a 3 (in some location)

- NOTE: more advanced players would likely avoid this guess, as it can’t possibly win. But it is a simple way to find out for sure it the digit 3 is included. An alternative approach is covered later.
Code is 1231 – Guess 4

- We know the code contains 1, 2 and 3. It does not contain 4 or 5. Do we need to try 6?

- 1122 response was 1-2. So three of the digits are 1 and 2
- 3333 response was 1-0. So one digit must be a 3
- That’s four digits…. so no need to try 6

- Let’s rearrange our first guess and try 1221
- The response is 3-0
The response to 1221 was 3-0 so three digits are in the correct location.

We know that the code also contains a 3.

Let’s try putting 3 in one location at a time, first 3221.

The response is 2-1.

So 3 is NOT the first digit.
Code is 1231 – Guess 6

- 1221 response was 3-0
- 3221 response was 2-1

- Let’s try 3 in the next location: 1321
- The response is 2-2

- So 3 is NOT the second digit
Code is 1231 – Guess 7

• 1221 response was 3-0
• 3221 response was 2-1
• 1321 response was 2-2

• Let’s try 3 in the next location: 1231
• The response is 4-0

• HURRAY! We found the code in 7 guesses.
Alternative sequence for 1231

Here’s a more effective sequence, but it requires slightly more reasoning, especially for guess 3.

1. 4455 => 0-0 (no 4 or 5 in code)
2. 1122 => 1-2 (contains 1,1,2 or 1,2,2 - in some order)
3. 1222 => 2-0 (contains 12_ _)  
4. 1213 => 2-2 (we know the digits, fix the order)
5. 1231 => 4-0!
QUESTION 4

Strategies for computer players
Are computers better at this?

- Do you think a computer program could be written that would perform better at this task than we do?

- Try this (optional, copy and paste into browser):
  http://csunplugged.mines.edu/mastermind/mastermind.cgi
A computer strategy

Here’s one *algorithm* (step-by-step process) a computer might use.

Remember there are 1296 possible codes!

1. Select a random code and ask the codemaker for response.
2. If the codemaker gives a winning response (i.e., 4-0), we have won. Stop.
3. Discard all elements which would no longer be possible under the codemaker’s response.
   - How would we know which ones are no longer possible?
4. Goto step 1
How does the computer eliminate codes?

Assume the secret code is 1231.

After a few guesses, assume the only possible codes that remain are:

- 1222
- 1322
- 1221
- 1231
- 1231

Computer randomly selects 1222.

What would the response be?
Eliminating Codes

• Right! The response would be 2-0

• So is 1222 correct? No!

• Which other codes could be correct, given that response?
Eliminating Codes

• Could the answer be 1322?

• Remember the computer guessed 1222.

• If correct code is 1322, then a “guess” of 1222 would get a “response” of 3-0 (3 digits in correct location, 1_22).

• BUT, response was 2-0, not 3-0. So remove 1322!
Eliminating Codes

- Could the answer be 1221?

- Remember the computer guessed 1222.

- If correct code is 1221, then a “guess” of 1222 would get a “response” of 3-0 (3 digits in correct location, 122_).

- BUT, response was 2-0, not 3-0. So remove 1221!
Eliminating Codes

• Could it be 1231?

• Remember the computer guessed 1222.

• If correct code is 1231, then a “guess” of 1222 would get a “response” of 2-0 (2 digits in correct location, 12__).

• Response was 2-0. So 1231 might be the right answer. Keep it!
Eliminating codes

1222 – eliminated (guessed, not correct)
1322 – eliminated (based on response)
1221 – eliminated (based on response)
1231 – still a possible answer

Only 1 possible answer remaining – so it must be 1231!
How well does this strategy perform?

- Using a computer to simulate many Computer vs Computer gameplays, here’s how frequent each number of guesses takes:

  Guesses in five tries or less ~85% of the time!
Can we do better?

• Making an “educated guess” about which code to choose (rather than just random):

Guesses in five tries or less ~97% of the time!

This is what the computer program demo uses.
Can we do even better?

- Minimax strategy (used in economics and AI applications) can win in 5 tries or less 100% of the time.
  - Downside: Very computationally expensive, especially when the game is extended to more digits
  - Downside: You can get a better average number of moves when you don’t impose a hard limit of 5 moves (think about this: why might it be so?)
Good for Computers, not Humans

- The computer can use *exhaustive search* – generate all possible combinations then eliminate them

- Would you want to try to do exhaustive search?

- Computer programs have been developed that are guaranteed to solve the Mastermind problem in at most 5 guesses (and can sometimes break the code in fewer guesses).